Probability: Events

Bayes' Rule

EXAMPLE:
Suppose we are given the following (hypothetical) information:

- *First born children have a 50% chance of being female.*
- *If the first child is a girl then the probability that the second child is a girl is* $\frac{1}{3}$.
- *If the first child is a boy then the probability that the second child is a girl is* 0.40.

In this situation, what is the probability that the first child is a female if the second child is a female?

We already know the answer using a tree diagram, now we will implement Bayes' Rule to get the solution. **Bayes' Rule** is given by the following formula:

$$P(\text{A}|\text{B}) = \frac{P(\text{A})P(\text{B}|\text{A})}{P(\text{A})P(\text{B}|\text{A}) + P(\text{A}^c)P(\text{B}|\text{A}^c)}$$

where A^c is a complement of event A.

First we define events A and B:

A={first child is female}
B={second child is female}

The given information gives us what we need for the Bayes' Rule formula:

- $P(\text{A}) = 0.5$ because *first born children have a 50% chance of being female.*
- $P(\text{A}^c) = 0.5$ since $P(\text{A}^c) = 1 - P(\text{A}) = 1 - 0.5 = 0.5$
- $P(\text{B}|\text{A}) = 1/3$ which we know from the given information *if the first child is a girl then the probability that the second child is a girl is* $\frac{1}{3}$.
- $P(\text{B}|\text{A}^c) = 0.40$ because A^c is the 'first male' event and we know that *if the first child is a boy then the probability that the second child is a girl is* 0.40.

Now we just substitute these values into the Bayes' rule formula and get

$$P(\text{A}|\text{B}) = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{4}{10}} = \frac{\frac{1}{6}}{\frac{1}{6} + \frac{4}{20}} \approx 0.45$$

The same result as with the tree diagram method.